

# Review and testing of the CropEvapoTranspirationDualCoeff SimComponent in SIMPLACE

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*Current documentation:*

[https://simplace.net/doc/simplace\\_modules/class\\_net.simplace.sim.components.evapotran.fao56.CropEvapoTranspirationDualCoeff.html#class\\_net.simplace.sim.components.evapotran.fao56.CropEvapoTranspirationDualCoeff](https://simplace.net/doc/simplace_modules/class_net.simplace.sim.components.evapotran.fao56.CropEvapoTranspirationDualCoeff.html#class_net.simplace.sim.components.evapotran.fao56.CropEvapoTranspirationDualCoeff)

*Main goal:*

- Calculate soil evaporation and crop transpiration at potential conditions

*Dependencies:*

- net.simplace.sim.components.util.helper.EquationsFAO56

## 1. SUMMARY

This SimComponent calculates the potential rates of crop transpiration and soil evaporation following the methods described in the FAO-56 bulletin (Allen et al., 1998). As the main goal of this SimComponent is to calculate these rates at potential conditions (i.e., full water supply), the reduction coefficients  $iK_r$  and  $iK_s$  should be kept equal to 1 at all times. For non-optimal conditions, it is recommended to calculate evapotranspiration rates using other SimComponents specially developed and tested for this purpose (e.g., SlimWater). To describe water-use by the crop modelers can inform crop-specific values for the  $cKcMin$ ,  $cKcbIni$ ,  $cKcbMid$  parameters of this SimComponent. It's also possible to prescribe the parameters to correct the atmospheric conditions ( $cCharacteristicMeanRelHumidity$ ,  $cCharacteristicWindspeed$  and  $cCropHeight$ ). However, its recommended to keep them as default values because it may increase model complexity without consistent gain in performance. Full description of this SimComponent functioning is given below as well as a discussion on the adaptations made to accommodate this method into SIMPLACE, the sources of uncertainties and limitations.

## 2. DESCRIPTION

Although called "*CropEvapoTranspirationDualCoeff*", this SimComponent does not strictly follow the calculation steps described in Chapter 7 of Allen et al. (1998). This was done in good reason as the main goal of this SimComponent was to fractionate the daily-step potential evapotranspiration

into crop transpiration and soil evaporation for dynamic crop models, and not solely for irrigation prescriptions. The main difference from the original method is that this SimComponent uses the crop development index (iDVS) and leaf area index (LAI) as proxy for the general scale of crop development specified in the FAO-56 bulletin (1: initial stage, 2: crop development stage, 3: mid-season stage, 4: late season stage). The input variables iDoSow, iDoHarvest and iDVS are used to control which set of equations will be applied at different crop development stage. While iDoSow and iDoHarvest are Boolean variables to inform the time-step when either sowing or harvesting happen (TRUE or FALSE), the iDVS is the crop development index that is used to indicate whether the crop has emerged or not (e.g. when DVS > 0). Another noticeable difference from the original dual coefficient method (Allen et al., 1998), is that it does not include a soil water balance routine to calculate the soil evaporation reduction factor (iK\_r). Instead, modelers can prescribe both reduction factor for soil evaporation (iK\_r) and crop transpiration (iK\_s). As the main goal of this SimComponent is to calculate soil evaporation and crop transpiration at potential conditions, it is recommended to keep the values of iK\_r and iK\_s as equal to one at all times. This was done to preserve modularity across SimComponents of SIMPLACE.

### 2.1 Calculation of soil evaporation

Soil evaporation is calculated in all time-steps by means of  $K_e$ , which is a function of the maximum evapotranspiration coefficient ( $K_{cmax}$ ) and wetted soil fraction.  $K_{cmax}$  is assumed as “1.2”, and can be corrected for the average conditions of wind speed, relative humidity and canopy height (Equation 1). In addition,  $K_{cmax}$  is constrained to not exceed  $K_{cb}+0.05$ , assuming that soil evaporation ( $K_e$ ) would correspond to approximately 5% of the total maximum evapotranspiration at high transpiration rates (Equation 2).

$K_{cmax} = 1.2 + (0.04*(u_2-2) - 0.004*(RH_{min} - 45))*(h/3)^{0.3}$	(1)
$K_{cmax} = \max(K_{cmax}, K_{cb}+0.05)$	(2)

where  $K_{cmax}$  is the maximum evapotranspiration coefficient (1.2) corrected by the mean conditions of wind speed at 2 meters height ( $u_2$ ), minimum air relative humidity ( $RH_{min}$ ) and crop height ( $h$ ) in the corresponding crop development stage. In the SimComponent the  $u_2$ ,  $RH_{min}$  and  $h$  can be prescribed, as `cCharacteristicWindspeed`, `cCropHeight` and `cCharacteristicMeanRelHumidity`, respectively. The “1.2” constant in Equation 1 is introduced by Allen et al. (1998) to represent the effects of increased aerodynamic roughness, albedo and wetting intervals. Allen et al. (1998) also propose the application of Equation 1 when  $u_2$ ,  $RH_{min}$  and  $h$  are possible to be determined for the corresponding stage of crop development proposed by FAO (initial, development, mid-season, or late-season). Furthermore, modelers must also observe the intervals of the variables in which Equation 1 should be applied:

- $1 \text{ m s}^{-1} < u_2 < 6 \text{ m s}^{-1}$
- $20\% < RH_{min} < 80\%$
- $0.1 \text{ m} < h < 10 \text{ m}$

Correcting  $K_{cmax}$  with Equation 1 may lead to unexpected results as it was only tested in the context of irrigation prescriptions (Allen et al., 1998), and not with dynamic crop models. Therefore, it is recommended to use the default values of  $u_2$ ,  $RH_{min}$ , and  $h$  to preserve the original value of  $K_{cmax}$ ,

unless substantial evidence is available to justify its application. Further discussion about Equation 1 is provided in the section 3 of this documentation.

After sowing day, a vegetation cover fraction ( $f_c$ ) is calculated to scale  $K_{cmax}$  for the fraction of soil area that can be directly wetted ( $f_{ew}$ ) by rainfall or irrigation events (Equation 3 and 4). A new variable is produced at this step, namely  $K_{eupper}$ , which represents the maximum soil evaporation considering the  $f_{ew}$ . The  $K_e$  value is finally obtained as the difference between  $K_{cmax}$  and  $K_{cb}$  multiplied by the soil moisture reduction factor ( $iK_r$ ), where  $K_e$  must not exceed the  $K_{eupper}$  (Equation 6).

$f_c = (\max(K_{cb} - K_{cmin}, 0.01) / (K_{cmax} - K_{cmin})) ^ (1 + 0.5 * h)$	(3)
$f_{ew} = \min(1-f_c, f_w)$	(4)
$K_{eupper} = K_{cmax} * f_{ew}$	(5)
$K_e = \min(iK_r * (K_{cmax} - K_{cb}), K_{eupper})$	(6)

where  $f_c$  is the vegetation cover fraction;  $K_e$  and  $K_{cb}$  are the soil evaporation coefficient and basal crop coefficient, respectively;  $K_{cmin}$  is the minimum  $K_c$  for a dry bare soil with no ground cover ( $\sim 0.15$ );  $f_c$  is the vegetation cover fraction;  $f_w$  is the fraction of soil surface wetted by irrigation or precipitation without crop cover;  $f_{ew}$  is the wetted soil fraction taking into account the vegetation cover ( $f_c$ );  $iK_r$  is the evaporation scaling factor dependent on the cumulative depth of water depleted from the topsoil;  $h$  is the mean plant height. Referential values for  $f_w$  can be found in Table 20 of Allen et al. (1998) for various irrigation types, whereas  $f_w=1$  is the default value of this SimComponent representing rainfall and sprinkler irrigation. Allen et al. (1998) also recommend limiting  $(K_{cb} - K_{cmin}) > 0.01$  for numerical stability.

## 2.2. Calculation of crop transpiration

Before sowing date,  $K_{cb}$  is set to zero, and after sowing it is set to the  $cK_{cbIni}$  parameter. When crop emerges ( $DVI > 0$ ), the procedure to determine  $K_{cb}$  starts by correcting the  $K_{cb}$  at full growth ( $K_{cbFull}$ ) with Equation 7. Note that this same equation is employed to correct  $K_{cmax}$  (Equation 1), but instead of “1.2”, users can provide the basal crop coefficient at the full canopy development stage ( $cK_{cbMid}$ ), here defined as the peak of LAI. As discussed earlier and in the section 3 of this documentation, we recommend using the default values of  $u_2$ ,  $RH_{min}$  and  $h$  when using this SimComponent. Equation 8 is then applied to calculate  $K_{cb}$  as a function of  $K_{cbFull}$ , LAI and  $cK_{cbIni}$ . Please note that Equation 8 is not completely similar to the Equation 97 presented by Allen et al. (1998) as it uses the  $K_{cbIni}$  instead of  $K_{cmin}$ . This was done in good reason to explicitly separate transpiration from soil evaporation, as discussed by DeJonge and Thorp (2017). Default value for  $cK_{cMin}$ ,  $cK_{cbIni}$  and  $cK_{cbMid}$  can be found in Table 1, but users could also source various crop-specific values of  $cK_{cbMid}$  from Allen et al. (1998) or studies aimed at determining basal crop coefficients.

$K_{cbFull} = cK_{cbMid} + (0.04*(u_2-2) - 0.004*(RH_{min} - 45))*(h/3)^{0.3}$	(7)
$K_{cb} = cK_{cbIni} + (K_{cbFull} - cK_{cbIni}) * (1 - \exp(-0.7 * LAI))$	(8)

where  $cK_{cbMid}$  is the basal crop coefficient at the full canopy development stage;  $K_{cbFull}$  is the  $cK_{cbMid}$  corrected by the mean conditions of wind speed at 2 meters height ( $u_2$ ), minimum air relative humidity ( $RH_{min}$ ) and crop height ( $h$ ) in the corresponding crop development stage; LAI is the leaf area index representing only healthy leaves that are active in vapour transfer;  $cK_{cbIni}$  is the

initial basal crop coefficient; and Kcb is the basal crop coefficient for the given time step. The “0.7” constant in Equation 8 is the canopy light extinction assumed by Allen et al. (1998) for natural, non-typical and non-pristine vegetation. Further discussion of this approach and its implications is provided in section 3 of this documentation.

Table 1. Default values, description and units for the different parameters of the “CropEvapoTranspirationDualCoeff” SimComponent. More information can be

Name in SimComponent	Description	Unit	Default
cCharacteristicMeanRelHumidity	Region's characteristic mean daily min relative humidity for mid/late growth season (RH_min)	%	45
cCharacteristicWindspeed	Region's characteristic wind speed at 2m during the mid growing season m s-1 (u2)	m/s	2
cCropHeight	Average crop height during mid/late season (h)	m	0
cKcMin	Minimum Kc for dry bare soil with no ground cover	dml	0.15
cKcbIni	Nominal Kcb value during initial growth stage	dml	0
cKcbMid	Nominal peak Kcb value obtained during mid season growth stage	dml	1
cWettedSoilFraction	Fraction of soil surface wetted by rain or irrigation	0-1	1
iK_r	Soil evaporation reduction coefficient dependent on the cumulative depth of water depleted from the topsoil	0-1	1
iK_s	Crop water stress factor (optional)	0-1	1

Soil evaporation (E) and crop transpiration (T) rates are then determined as the product between the reference crop evapotranspiration (ET0) and the corresponding Ke and Kcb coefficients. As this SimComponent was initially developed to simulate soil evaporation and transpiration at potential conditions, it is recommended to keep the iK\_r and iK\_s equals to 1. To calculate actual evapotranspiration rates, other SimComponents developed and tested specially for non-optimal conditions should be used (e.g. SlimWater). In addition, changing these parameters may modify the definition of the outputs, and should be done with extreme caution (see section 3). The outputs rates of potential soil evaporation (PotentialSoilEvapCrop) and crop transpiration (PotentialTranspiration) are expressed by Equations 9 to 11. If needed, modelers can also output KcMax, Kcb, Ke, KeUpper coefficients and ET-related variables (see Component Variables Table).

PotentialTranspiration = Kcb * ET0	(9)
PotentialSoilEvapCrop = Ke * ET0	(10)
ETC = PotentialTranspiration + PotentialSoilEvapCrop	(11)

where PotentialTranspiration is the potential crop transpiration, explicitly omitting the soil reduction factor (iK\_s); PotentialSoilEvapCrop is the potential soil evaporation, however, only when Ke is calculated with an iK\_r=1; ETC is the potential evapotranspiration.

### 3. LIMITATIONS AND UNCERTAINTIES

This section presents some of the limitations and uncertainties of this SimComponent, that can be viewed as opportunities for future improvements. Part of these lies on the assumptions that had to be made to adapt the FAO dual coefficient method into SIMPLACE. Firstly, as SIMPLACE is mainly used for dynamic simulations of crop growth, the *iDoSow*, *iDoHarvest* and *iDVS* variables were introduced for triggering initialization and transition processes throughout time. Secondly, the LAI is used as a proxy for crop development in Equation 8 instead of using the 4-stage scale of crop development defined by FAO (initial, crop development, mid-season, late season). This was adopted in good reason to avoid the challenge of converting development scales from other crop models into to the FAO-scale, that is used to interpolate the values of *Kcb* over time. However, some implications also arise from this assumption as discussed in section “Equation 8” below. We also recommend keeping the default values of *RH\_min* (“*cCharacteristicMeanRelHumidity*”), *u2* (“*cCharacteristicWindspeed*”), and *h* (“*cCropHeight*”, Table 1) when using this SimComponent, unless substantial evidence is provided to correct *Kc* values to the atmospheric conditions. It also important to note that this SimComponent was initially intended to simulate potential soil evaporation and crop transpiration. Therefore, using it for actual rates (i.e.,  $iK_r < 1$ ;  $iK_s < 1$ ) should be avoided without further testing. More information about the equations of this SimComponent can be found in literature cited in Table 2.

Table 2. Equations used in the DualCoefficient SimComponent and the corresponding literature source.

Equation Number	Literature
1	Equation 72 of Allen et al. (1998)
2	
3	Equation 76 of Allen et al. (1998)
4	Equation 75 of Allen et al. (1998)
5	Equation 71 of Allen et al. (1998)
6	
7	Equation 99 of Allen et al. (1998)
8	Equation 6 of DeJonge and Thorp (2017), adapted from Equation 97 of Allen et al. (1998)
9	Equation 69 of Allen et al. (1998)
10	
11	

### 3.1 Reduction coefficients ( $iK_s$ and $iK_r$ ) and the ETC definition

Both  $iK_r$  and  $iK_s$  factors are used in this SimComponent to scale the potential soil evaporation and potential transpiration rates, and they should be essentially derived from a soil water balance routine. Therefore, when either  $iK_r$  or  $iK_s$  are below one, the outputs of this SimComponent no longer correspond to the potential rate of atmospheric demand. Furthermore, attention should be taken by modelers who seek to compare the potential evapotranspiration (ETP) simulated by this SimComponent, defined as the sum of potential soil evaporation and the potential

crop transpiration, with measurements of crop evapotranspiration (ETC). The latter is introduced by Allen et al (1998) as the ET rate for crops grown in large fields under excellent agronomic and soil water conditions, which implies that  $iK_s$  is always nearly 1 at these conditions. However,  $iK_r$  cannot be equal to 1 at all times, otherwise the crop would suffer from severe waterlogging as  $iK_r=1$  implies in saturated soil, violating the principle of “excellent agronomic and soil water conditions” defined by Allen et al. (1998), unless under special conditions (e.g. flooded rice). Therefore, this SimComponent can only produce ETC outputs, per-se, when only  $iK_s = 1$  and  $iK_r$  is dynamically provided by a soil water balance routine.

### 3.2 Equation 8

Allen et al. (1998) introduced an equation to calculate the  $K_{cbFull}$  as a function of LAI for annual types of vegetation that are either natural or in a non-pristine (see Equation 97 of Allen et al. (1998)). This equation is recommended for crops with sparse canopy or under the effect of some type of environmental stress on growth. However, this equation can also be used to replace the FAO scale of crop development required to interpolate  $K_c$  by using LAI as a proxy for canopy development. Furthermore, to explicitly separate transpiration from soil evaporation the “ $K_c$  min” parameter of this equation should be replaced by the minimum  $K_{cb}$  (or the “ $K_{cbIni}$ ” parameter), as proposed by DeJonge and Thorp (2017). This new equation is employed in SIMPLACE (Equation 8) to mimic the trapezoidal pattern of  $K_{cb}$  over time in a dynamic way.

However, it should also be noted that while Equation 8 employs the widely used Beer law principle to compute transpiration, the vegetation cover fraction ( $f_c$ ) used to calculate soil evaporation use a different approach (Equation 3). Furthermore, it might be difficult to identify the effect that Equation 8 may exert over soil evaporation, due to the dependency of Equation 3 on Equation 8, especially under different scenarios of canopy height ( $h$ ) and energy “extinction” coefficient (“0.7” term of Equation 8).

Modelers should also be cautious when applying Equation 8 as the term “ $1 - \exp(-0.7 * LAI)$ ” may never reach to the unit for their desired crop. For example, when the “0.7” constant is assumed, the “ $1 - \exp(-0.7 * LAI)$ ” would only approximate to 1 when LAI is greater than  $6 \text{ m}^2 \text{ m}^{-2}$ , and is 0.88 when LAI is  $3 \text{ m}^2 \text{ m}^{-2}$ . This means that if the crop is not able to reach at this LAI threshold of  $6 \text{ m}^2 \text{ m}^{-2}$ , simulations will never reach the maximum  $K_{cb}$  even at potential growth conditions. Another implication of this adaptation is that it assumes that the initial and the late crop developmental stages have the same  $K_c$  coefficients, whereas this is observed for a very few crops (e.g., Table 12 and Table 17 of Allen et al. (1998)). While Allen et al. (1998) do not discriminate whether the green LAI or total LAI (green + senesced) should be used, we can hypothesize these differences are associated to the distinct physiological status of the crop at the initial (vegetative growth) and late (senescence) stages. Although its plausible to assume that green LAI is the major canopy component controlling transpiration, the dry canopy structure standing on the field during (and after) senescence may still play a role in the surface roughness, and hence on soil evaporation. Another explanation for the discrepancy of  $K_c$  values at initial and late stages reported by Allen et al. (1998) could be due to the non-equal duration in days of initial and late stages (e.g., Table 11 of Allen et al. (1998)). While many open questions remain from this exercise, it’s still difficult to measure, quantify and test these effects, especially at field scale.

### 3.3 Correction for characteristic $RH_{min}$ , $u_2$ and $h$

Please note that Equation 1 is introduced by Allen et al. (1998) to correct crop coefficients under climate conditions different than those of  $RH_{min} = 45\%$ ,  $u_2 = 2 \text{ m s}^{-1}$  and  $h = 3 \text{ m}$ . Although modelers can provide daily or average values for  $u_2$ ,  $RH_{min}$ , and  $h$  for a given crop developmental stage, this practice is not encouraged when using this SimComponent with dynamic crop models. Firstly, it may be difficult to establish a direct relationship between the scales of crop development provided by Allen et al. (1998) and the one employed in the crop model. As a result, the definition of the time period for averaging the  $u_2$ ,  $RH_{min}$ , and  $h$  conditions become very difficult. Secondly, the intervals domain of  $u_2$ ,  $RH_{min}$ , and  $h$  in that Equation 1 was tested may not be generally met to all conditions, especially if daily values of  $u_2$ ,  $RH_{min}$ , and  $h$  are provided. Finally, when modelers use  $K_c$  values measured in a field experiment, it may already incorporate the  $u_2$ ,  $RH_{min}$ , and  $h$  characteristics for that given condition, thus Equation 1 is not necessary at this situation and its use may lead to unexpected results as the referential  $K_c$  is not anymore at Allen et al. (1998) standard conditions (e.g.,  $RH_{min} = 45\%$ ,  $u_2 = 2 \text{ m s}^{-1}$  and  $h = 3 \text{ m}$ ).

Furthermore, Equation 1 was seldomly validated as field experiments aimed to determine crop coefficients across different climates are rare. Shahrokhnia and Sepaskhah (2013) tested the Single and dual crop coefficients methods for wheat and maize in a semi-arid region. Even strictly following the Allen et al. (1998) methodology, they had to adapt the Equation 1 to better represent the  $K_c$  values observed for maize and wheat. Yet, the FAO procedure for single crop coefficient showed better predictions on a daily basis, although the dual crop coefficient method was more accurate on seasonal scale.

We tested this Equation against eddy covariance measurements taken in two fully-irrigated maize field experiments in Nebraska, USA, from 2003 to 2012. This dataset was obtained from the FLUXNET database (<https://fluxnet.org/>), indexed as US-Ne1 and US-Ne2, and are located within 1.6 km of each other. US-Ne1 grows maize whereas US-Ne2 is maize-soybean rotation, and both experiments are fully-irrigated by a central pivot system. Canopy height ( $h$ ) and leaf area index (LAI) measurements were taken over time by measuring tape ( $h$ ) and destructive method (LAI). To obtain continuous values of  $h$  and LAI for each trial, we used a gauss function to interpolate LAI values and a flexible sigmoidal function for  $h$  (Yin et al., 2003). We then assumed the  $K_{cFull}$  as the ratio between  $ET_C/ET_0$  only when  $LAI > 0.95 \cdot \text{Max\_LAI}$ . Please note that the average values for  $RH_{min}$ ,  $u_2$  and  $h$  considered in this analysis also only considered the period assumed for  $K_{cFull}$  (when  $LAI > 0.95 \cdot \text{Max\_LAI}$ ). Where  $ET_0$  was computed by the Penman-Monteith method and the  $\text{Max\_LAI}$  was the maximum LAI observed in each season. Figure 1 show that the response of  $K_{cFull}$  to  $RH_{min}$ ,  $u_2$  and  $h$  in these field trials are quite scattered and might not always be represented by Equation 1 (e.g. for  $RH_{min}$ ). The range of predictions of  $K_{cFull}$  using this equation was also narrower and lower ( $K_{cFull} = 1.18 \pm 0.31$ ) than the observed ( $K_{cFull} = 1.36 \pm 0.67$ ) (Figure 2). At these conditions, using the atmospheric correction factors (Equation 1) increased model complexity without improving  $K_c$  simulations (Figure 2). Therefore, unless substantial evidence is available to justify the use of Equation 1 for correcting  $K_c$ , modelers should keep the default values for  $u_2$ ,  $RH_{min}$ , and  $h$  when coupling this SimComponent with dynamic crop models.

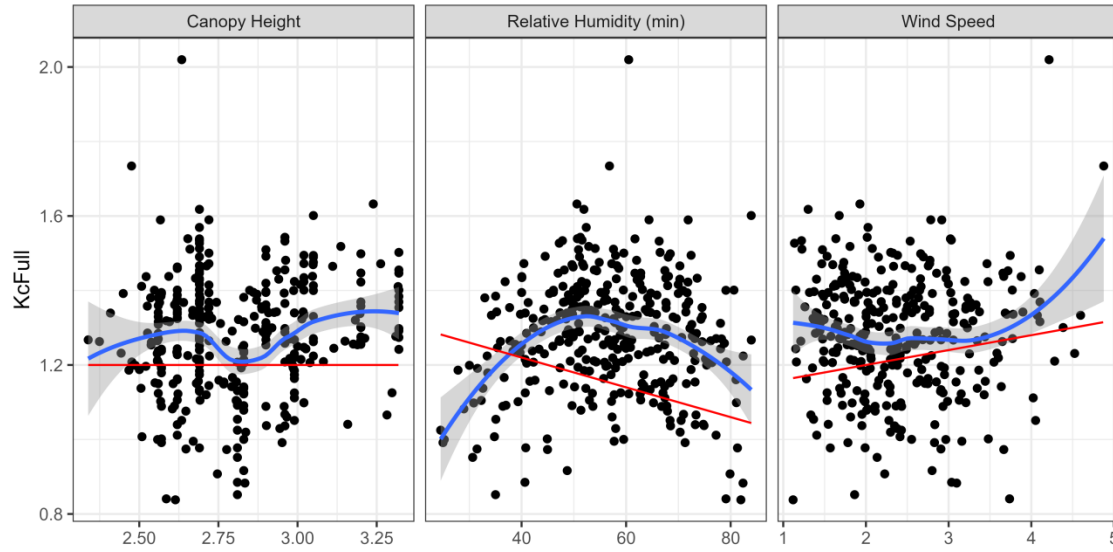


Figure 1. Crop coefficient at full canopy development stage ( $Kc_{Full}$ ) observed in 17 fully-irrigated maize seasons in Nebraska, USA (solid circles), the response of the atmospheric correction proposed by Allen et al. (1998) in Equation 1 and 7 (red lines), and the spline regression adjusted to observed data (blue line) as a function of canopy height ( $h$ ), minimum relative air humidity ( $RH_{min}$ ) and wind speed ( $u_2$ ). The red line was derived assuming the  $Kc_{Mid}=1.20$  for maize following the FAO bulletin (Tab 12) and the average of  $RH_{min}$  (56.7%),  $u_2$  (2.38 m s<sup>-1</sup>) and  $h$  (2.81 m) for the maize seasons when the sensitivity range was not applied for each parameter (e.g. the leftmost panel assumes  $RH_{min}=56.7\%$ ,  $u_2=2.38$  m s<sup>-1</sup>, for the  $h$  range between 2.3 to 3.3 m).



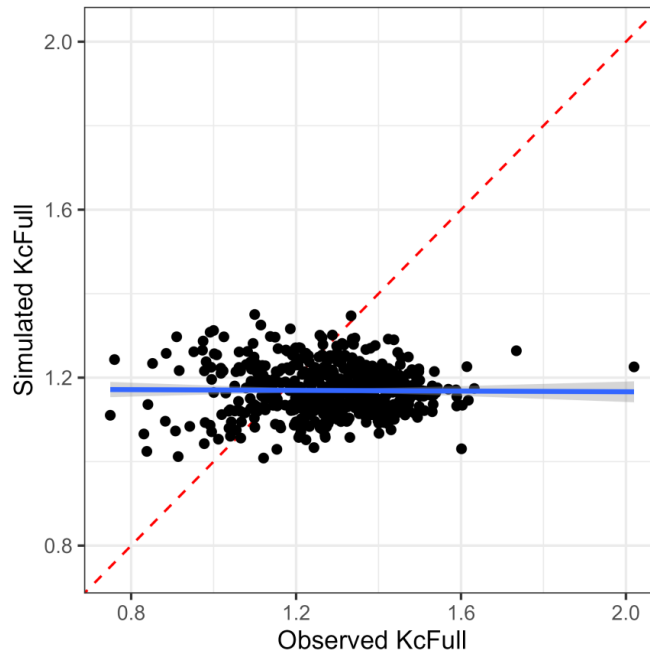


Figure 2. Comparison between observed and simulated KcFull estimates in 17 fully-irrigated maize seasons in Nebraska (USA) following the FAO atmospheric correction approach (Equation 7). Results were obtained assuming the KcMid=1.20 proposed in Tab 12 of Allen et al. (1998) and the corresponding average of RH\_min, u2 and h for each season.

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